Solutions of Mock JEE Advanced – 2 | Paper - 2 | JEE 2024

PHYSICS

Section – 1

1.(ACD)

(i) $\rho = \alpha$ for $r \le \dfrac{R}{2}$

(ii) $\rho = 2\alpha\left[1 - \dfrac{r}{R}\right]$ for $\dfrac{R}{2} \le r \le R$

Charge in the region $r \le \dfrac{R}{2}$

$$Q_a = \alpha.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3$$

$$Q_a = \alpha.\frac{4}{3}\pi\frac{R^3}{8} \Rightarrow Q_a = \frac{\alpha\pi R^3}{6}$$

Charge in the region of the sphere $\dfrac{R}{2} \le r \le R$

$$Q_b = \int \rho dv$$

$$= \int 2\alpha\left[1 - \frac{r}{R}\right].4\pi r^2 dr$$

$$Q_b = 8\pi\alpha\int_{\frac{R}{2}}^{R}\left(r^2 - \frac{r^3}{R}\right)dr$$

$$Q_b = \frac{11\alpha\pi R^3}{24}$$

Total charge $Q = \dfrac{\alpha\pi R^3}{6} + \dfrac{11\alpha\pi R^3}{24}$

$$Q = \alpha\pi R^3\left[\frac{1}{6} + \frac{11}{24}\right]$$

$$Q = \frac{15}{24}\alpha\pi R^3 \Rightarrow \alpha = \frac{8Q}{5\pi R^3}$$

Apply gauss theorem in the region $r \le \dfrac{R}{2}$

$$E.4\pi r^2 = \frac{q}{\varepsilon_0}$$

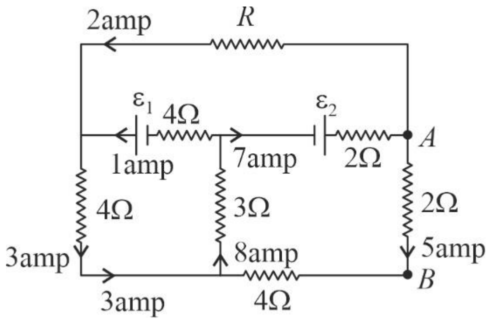
$$E.4\pi r^2 = \frac{1}{\varepsilon_0}\frac{4\pi}{3}r^3\alpha$$

$$E = \frac{\alpha r}{3\varepsilon_0}$$

$$E = \frac{8Qr}{15\pi\varepsilon_0 R^3}$$

2.(ABCD)

By applying Kirchhoff's law



$\varepsilon_1 = 40V$

$\varepsilon_2 = 68V$

$R = 9\Omega$ and current through $4\Omega = 5\,amp$.

3.(BC)

For the bubble, PV = const.

$$\therefore \qquad \frac{dV}{V} = \frac{-dP}{P}$$

$$\frac{dV}{V} = \frac{3dr}{r}, dP = \frac{4T}{R}, P \approx P_\circ$$

$$\therefore \qquad dr = \frac{4Tr}{3P_0R}$$

4.(ABD)

At a distance h above the sheet

$E = E_{sheet} + E_{slab}$

$$= \frac{-\sigma}{2\varepsilon_0} + \frac{\rho D}{2\varepsilon_0} = \frac{\rho D - \sigma}{2\varepsilon_0}$$

At a distance h below the top surface of slab

$E_{slab} = \dfrac{\rho(D-2h)}{2\varepsilon_0}$

$E = E_{sheet} + E_{slab} = \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\rho(D-2h)}{2\varepsilon_0}$

$= \dfrac{\sigma+\rho(D-2h)}{2\varepsilon_0}$

At a distance h below the bottom surface of the slab = $\dfrac{-\sigma}{2\varepsilon_0}+\dfrac{\rho D}{2\varepsilon_0}=\dfrac{\rho D-\sigma}{2\varepsilon_0}$

5.(AD) Shift in pattern $=\dfrac{D}{d}\times$(change in path difference) $=\dfrac{D}{d}[\mu l - l]=\dfrac{D}{d}(\mu-1)l=\dfrac{\omega}{\lambda}(\mu-1)l$

6.(ABCD)

$T_A=300K$, $T_C=900K$, $T_B=100K$

$\therefore \quad W_{BC}=nR\Delta T=1\times R\times 800=800R$

$\Delta U_{CA}=n\times\dfrac{3R}{2}\times\Delta T=1\times\dfrac{3R}{2}\times(-600)=-900R$

For $A\to B, P\times(PV)=$ constant

$\Rightarrow \quad PV^{\frac{1}{2}}=$ const.

$\therefore \quad W_{AB}=\dfrac{1\times R\times(200)}{\left(\frac{1}{2}-1\right)}=-400R$

Section – 2

7.(A) Magnetic field at centre of circular loop by the bar magnet = $\dfrac{\mu_0}{4\pi}\dfrac{2M}{x^3}$

Magnetic flex through circular loop, $\phi=B\pi a^2$

Induce emf in circular loop, $e=-\dfrac{d\phi}{dt}$ and induce current in circular loop $i=\dfrac{e}{R}=-\dfrac{d\phi}{(dt)R}=\dfrac{3\mu_0 Ma^2v}{2x^4R}$

8.(A) $\dfrac{1}{2}mv^2=\dfrac{hc}{\lambda}-\phi$

In order to describe a circle

$\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{d^2}=\dfrac{mv^2}{d}$

$$\Rightarrow \quad mv^2 = \frac{e^2}{4\pi\varepsilon_0 d}$$

$$2\left(\frac{hc}{\lambda} - \phi\right) = \frac{e^2}{4\pi\varepsilon_0 d}$$

$$\Rightarrow \quad \frac{hc}{\lambda} = \frac{e^2}{8\pi\varepsilon_0 d} + \phi$$

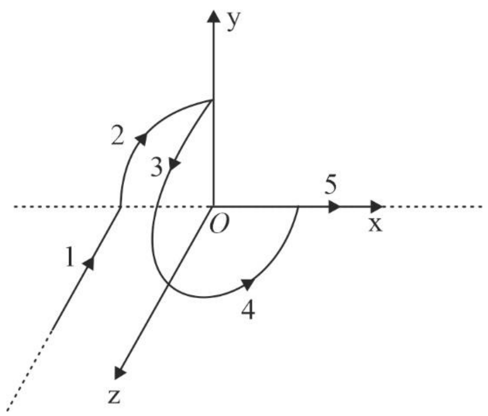
$$\Rightarrow \quad \lambda = \frac{hc}{\frac{e^2}{8\pi\varepsilon_0 d} + \phi} = \frac{8\pi\varepsilon_0 hcd}{e^2 + 8\pi\varepsilon_0 d\phi}$$

9.(B) $LC = \frac{\text{pitch}}{\text{circular scale divisions}} = \frac{0.5}{50} mm = 0.01$

Negative zero error = $-5\times(0.01)mm = -0.05mm$

Measured value = $0.5\,mm + 25\times 0.01 - (-0.05)\,mm = 0.8mm$

10.(A) $\vec{B}_0 = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_6$



$$\vec{B}_0 = \frac{\mu_0 i}{4\pi R}\left(-\hat{j}\right) + \frac{\mu_0 i}{8R}\left(-\hat{k}\right) + \frac{\mu_0 i}{8R}\hat{i} + \frac{\mu_0 i}{8R}\hat{j} + 0$$

$$\vec{B}_0 = \frac{\mu_0 i}{8R}\left[\hat{i} - \hat{k} + \hat{j}\left[1 - \frac{2}{\pi}\right]\right]$$

Section – 3

1.(2) At certain instant, $v = \sqrt{v_x^2 + v_y^2}$

Rate of change of speed is given by

$$\frac{dv}{dt} = \frac{2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}}{2\sqrt{v_x^2 + v_y^2}}$$

$$= \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}} \qquad \because \qquad \frac{dv_x}{dt} = a_x \text{ and } \frac{dv_y}{dt} = a_y$$

$$= \frac{3\times 2 + 4\times 1}{\sqrt{3^2+4^2}} = 2\,m/s^2$$

2.(4) $${}^{238}_{92}U \rightarrow {}^{214}_{84}Po + 6\alpha + ne^-$$

$$92 = 84 + 12 - n$$

$$n = 4$$

3.(9) $Ig(G+R_1) = 3$ …(i)

$Ig(G+R_1+R_2) = 15$ …(ii)

$Ig(G+R_1+R_2+R_3) = 150$ …(iii)

Solving, we get $R_1 = 2.98\,k\Omega$

$R_2 = 12\,k\Omega$

$R_3 = 135\,k\Omega$

4.(7) $$P = \mu_0^x m^y \omega^z c^u$$

$$[P] = \left[MLT^{-2}\right]\left[LT^{-1}\right]$$

$$[\mu_0] \rightarrow \left[MLT^{-2}A^{-2}\right], [m] \rightarrow \left[AL^2\right]$$

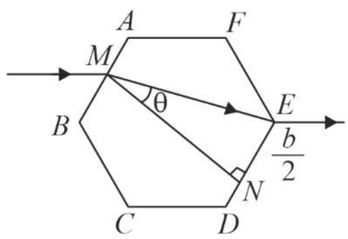
$$[\omega] \rightarrow \left[T^{-1}\right], [c] \rightarrow \left[LT^{-1}\right]$$

On solving we get

$$x = 1, y = 2, u = -3, z = 4$$

5.(9) For light beam to emerge parallel to the incident direction it must strike the face DE which is parallel to face *AB*. It means incident ray at *M* must go to *E* (or below it). Let side length of hexagon be *b*.

$$MN = \sqrt{3}b$$



$$\therefore \qquad ME = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\sqrt{3}b\right)^2} = \frac{\sqrt{13}}{2}b$$

Using Snell's law

$$1.\sin 30^\circ = \mu \sin\theta$$

$$\Rightarrow \qquad \frac{1}{2} = \mu\frac{1}{\sqrt{13}} \qquad \Rightarrow \qquad \mu = \sqrt{\frac{13}{4}}$$

6.(6) In the figure, we have shown a charge q at the centre of one open end of a cylinder. Note that the other end is closed. Now, take another identical cylinder and join open end with the open end of the first cylinder. Now, we have closed surface of two cylinders. The electric flux through this closed surface, according to Gauss's theorem is q/ε_0 where ε_0 is electric permittivity of space. Then, from the symmetry, we can say that flux through surface of any one of the two joined surface will be $\dfrac{q}{2\varepsilon_0}$

7.(3) Half the oscillation is completed with one spring and the other half with the other spring. Hence,

$$T = \frac{T_1}{2} + \frac{T_2}{2}$$

$$= \frac{2\pi\sqrt{\dfrac{M}{k}}}{2} + \frac{2\pi\sqrt{\dfrac{M}{4K}}}{2}$$

$$= \pi\sqrt{\frac{M}{k}}\left[1 + \frac{1}{2}\right]$$

$$= \frac{3\pi}{2}\sqrt{\frac{M}{k}}$$

8.(6) By linear momentum conservation

Impulse (J) = mV. By angular momentum conservation, angular impulse = $J\dfrac{l}{2} = I\omega$

So $mv\dfrac{l}{2} = I\omega \; or \; \omega = \dfrac{mvl}{2I} = \dfrac{mvl}{2\left(\dfrac{ml^2}{12}\right)} = \dfrac{6v}{l} = 6rad/s$

CHEMISTRY

Section – 1

1.(AB) Molecular mass = $2\times73 = 146$

Element	Percentage	Relative no. of atoms	Simple Ratio
C	49.3	4.1	1.5 × 2
H	6.84	6.84	2.5 × 2
O	43.86	2.74	1 × 2

(A) So, Empirical formula is $C_3H_5O_2$

(B) Molecular formula = $2\times C_3H_5O_2$

$C_6H_{10}O_4$

One of the isomer is dicarboxylic acid, so it reacts with two moles of Grignard reagent

(C) $HOOC-CH_2-\underset{|}{\overset{COOH}{CH}}-CH_2-CH_3$

Optically active

(D) Empirical formula is $C_3H_5O_2$

2.(ABCD)

(A) As the chain becomes more branched the top-like rotation of one carbon atom with respect to another becomes more hindered

(B) Heteronuclear diatomic molecules have higher entropy than homonuclear diatomic molecules because they possess a more random distribution of matter

(C) For isothermal process $T_2 = T_1$

$$\Delta S = nR\ln\frac{V_2}{V_1} = -nR\ln\frac{P_2}{P_1}$$

(D) For adiabatic process $dq = 0, \Delta S = 0$

Hence isoentropic

3.(AB)

(A) $Na_2B_4O_7.10H_2O \xrightarrow{\Delta} 2NaBO_2 + B_2O_3 + 10H_2O$

(B) Even with fluorine, Boron forms covalent BF_3

(C) Only 2 Boron are involved in Back Bonding

(D) In borax two boron are sp^2 hybridised and two are sp^3.

4.(BC)

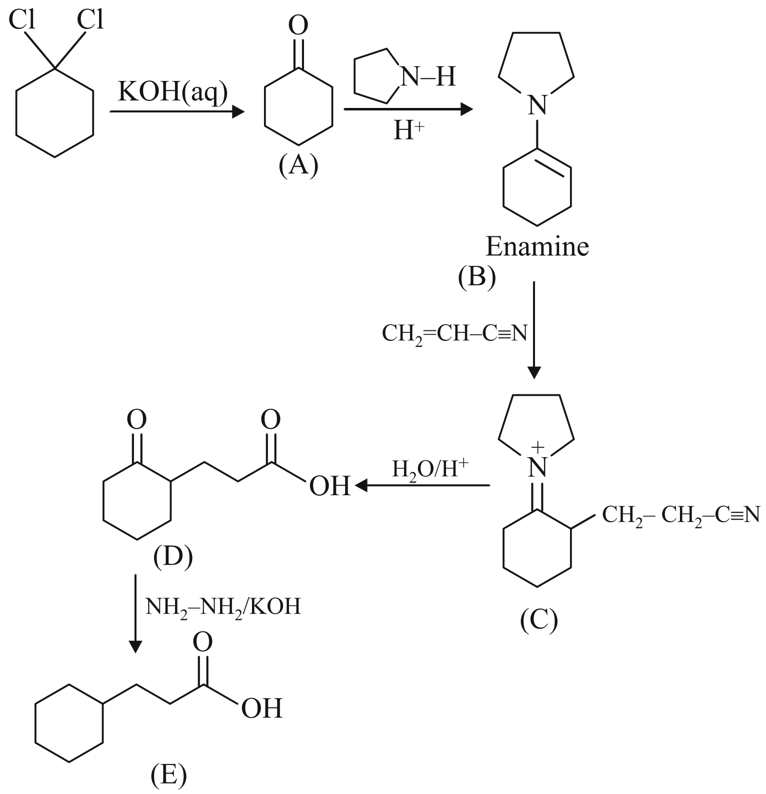
$$FeS + \frac{3}{2}O_2 \rightarrow FeO + SO_2$$

$$FeO + SiO_2 \rightarrow FeSiO_3$$

$$CuFeS_2 + O_2 \rightarrow Cu_2S + FeS + SO_2$$

Fe has more affinity for oxygen than Cu.

5.(ABD)



(A) Compound E is carboxylic acid

(B) A gives aldol reaction with dilute NaOH

(C) D has 6 $(-CH_2-)$ groups

(D) B is enamine which shows imine enamine tautomerism

6.(AB)

(A) Dacron is crease resistant fibre

(B) Polystyrene is a thermoplastic

(C) Natural rubber behaves as elastomers

(D) Polyester is a copolymer

Section – 2

7.(A) Number of atoms of $A = 8 \times \frac{1}{8} = 1$

Number of B atoms = $5 \times \frac{1}{2} = 5/2$

$A : B = 1 : \frac{5}{2}$

$= 2 : 5$

So formula is A_2B_5

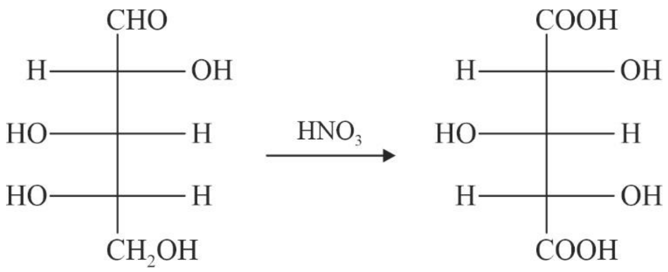
8.(C) $\underset{\text{(hot and conc.)}}{6NaOH + 3Cl_2} \longrightarrow 5NaCl + \underset{(X)}{NaClO_3} + 3H_2O$

$2NaClO_3 \longrightarrow 2NaCl + 3O_2$ (Paramagnetic)

9.(B) $Pb^{+2}, Zn^{+2}, Cd^{+2}, Al^{+3}, Cu^{+2} \xrightarrow{H^+/H_2S} PbS\downarrow + CuS\downarrow + CdS\downarrow + Al^{+3}$

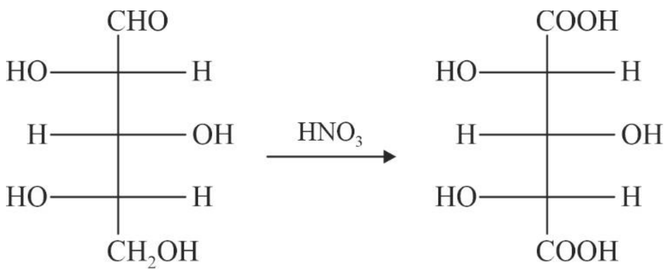
and Zn^{+2} present in solution $\xrightarrow{\text{Excess of NaOH}} \underset{\text{(Soluble)}}{\left[Al(OH)_4\right]^- + \left[Zn(OH)_4\right]^{2-}}$

10.(A) (A) Only (A) and (D) are L sugar



Optically active

(B)



Meso

Section – 3

1.(5) $AgI_{(S)} \rightleftharpoons Ag^{+}{}_{(Aq.)} + I^{-}{}_{(Aq.)}$

$K_{sp} = 1.2 \times 10^{-17}$

$Ag^{+}{}_{(Aq.)} + 2CN^{-} \rightleftharpoons Ag(CN)_2^{-}$

$K_c = 7.58 \times 10^{19}$

Let '*a*' mole of AgI be dissolved in CN^{-} solution,

	$AgI_{(s)}$	$+2CN^{-}$	$\rightleftharpoons Ag(CN)_2^{-}$	$+I^{-}$
Initial moles	1	1	0	
Moles at Equilibrium	$(1-a)$	$(1-2a)$	a	a

$$K_{eq} = \frac{\left[Ag(CN)_2\right]^{-} \times \left[I^{-}\right]}{\left[CN^{-}\right]^2}$$

$$K_{eq} = K_{sp} \times K_f \qquad \text{...(i)}$$

$$\frac{a^2}{(1-2a)^2} = 1.2 \times 10^{-17} \times 7.58 \times 10^{19}$$

$$\frac{a^2}{(1-2a)^2} = 9.09 \times 10^{2}$$

$$\frac{a}{1-2a} = 3 \times 10^{1}$$

$a = 30 - 60a$

$61a = 30$

$a = \frac{30}{61}$

$\simeq 0.5$ mole

$a = 5 \times 10^{-1}$

So, *x* is 5

2.(7) $\Delta T_f = ik_f m$

$$i = \frac{\Delta T_f}{K_f m}$$

$$m = \frac{19.5 \times 1000}{78 \times 500} = 0.5$$

$$i = \frac{1.22}{1.86 \times 0.5}$$

$$i = 1.3118$$

Let α be the degree of dissociation of CH_2FCOOH

$$\begin{array}{cccccc} CH_2FCOOH & \rightleftharpoons & CH_2FCOO^- & + & H^+ \\ C & & O & & O \\ C - C\alpha & & C\alpha & & C\alpha \end{array}$$

$$i = \frac{C + C\alpha}{C}$$

$$1.3118 = 1 + \alpha$$

$$\alpha = .3118$$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$$K_a = \frac{(.3118)^2 \times .5}{1 - .3118}$$

$$K_a = 0.07$$

$$K_a = 7 \times 10^{-2}$$

So value of y is 7

3.(9)

$$\begin{array}{lcccccc} & HX & \rightleftharpoons & H^+ & + & X^- \\ \text{Initially} & 0.1 & & 0 & & 0 \end{array}$$

Let degree of dissociation of HX be α

at equilibrium $(0.1 - 0.1\alpha)$ $\quad 0.1\alpha \quad 0.1\alpha$

$$\wedge_m = \frac{\kappa \times 1000}{C}$$

$$= \frac{5 \times 10^{-6} \times 1000}{0.1}$$

$$= 0.05 \, S cm^2 \, mol^{-1}$$

$$\alpha = \frac{\wedge_m}{\wedge_M^\infty} = \frac{0.05}{(0.04 + 0.01) \times 10^4}$$

$$= \frac{.05}{.05 \times 10^4} = 10^{-4}$$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$$1-\alpha \simeq 1$$

$$K_a = C\alpha^2$$

$$K_a = .1\times\left(10^{-4}\right)^2$$

$$K_a = 10^{-9}$$

$$pK_a = 9$$

4.(9) $XeF_4 + O_2F_2 \xrightarrow{143K} \underset{(P)}{XeF_6} + O_2$

$$4XeF_6 + 36NaOH \xrightarrow{143K} 3Na_4XeO_6 + Xe + 24NaF + 18H_2O$$

4 mol of XeF_6 required 36 mol of NaOH

So 1 mol of XeF_6 required 9 mol of NaOH

5.(0) $NH_4NO_2 \longrightarrow \underset{(gas)}{N_2} + 2H_2O$

$$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \pi 2px^2 = \pi 2p_y^2 < \sigma 2p_z^2 < \pi^* 2p_x = \pi^* 2p_y < \sigma^* 2p_z$$

No electrons present in $\pi^* 2p_x$ and $\pi^* 2p_y$

6.(8) $CH_3-\underset{\substack{| \\ OH}}{CH}-CH_2-CH=CH-CH_3 \xrightarrow{Br_2/CCl_4} CH_3-\underset{\substack{| \\ OH}}{CH}-CH_2-\underset{\substack{| \\ Br}}{CH}-\overset{\substack{Br \\ |}}{CH}-CH_3$

$$\xrightarrow{Conc.H_2SO_4,\Delta} CH_3-CH=CH-\underset{\substack{| \\ Br}}{\overset{*}{CH}}-\underset{*}{\overset{\substack{Br \\ |}}{CH}}-CH_3$$

Total number of stereoisomers are 8

7.(4) $CH_3-CH_2-CH_3 \xrightarrow{Br_2/h\nu} \underset{(A)}{CH_3-\underset{\substack{| \\ Br}}{CH}-CH_3} \xrightarrow{aq.KOH} \underset{(B)}{CH_3-\underset{\substack{| \\ OH}}{CH}-CH_3}$

$$\xrightarrow{Na} CH_3-\underset{\substack{| \\ ONa}}{CH}-CH_3$$

$$CH_3-CH_2-CH_3 \xrightarrow{Br_2/h\nu} \underset{\substack{|\\ Br \\ (A)}}{CH_3-CH-CH_3} \xrightarrow{\text{alc.KOH}} CH_3-CH=CH_2$$

$$\xrightarrow{NBS} Br-CH_2-CH=CH_2 \xrightarrow[\text{ONa}]{CH_3-\underset{|}{C}H-CH_3} \begin{matrix}CH_3\\CH_3\end{matrix}\!\!>CH-O-\overset{*}{C}H_2-CH=\overset{*}{C}H_2$$

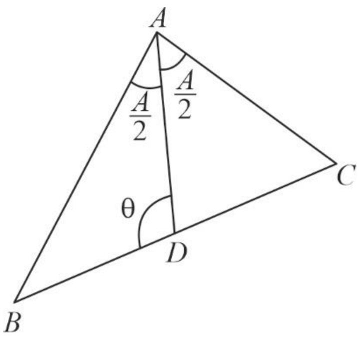
8.(8) $$H-\underset{\substack{\|\\ O}}{C}-\underset{\substack{\|\\ O}}{C}-H + H-\overset{\substack{O\\ \|}}{C}-\underset{\substack{\|\\ O}}{C}-\underset{(2)}{CHDOH} + H-\overset{\substack{O\\ \|}}{C}-\underset{\substack{\|\\ O\\ (2)}}{C}-CHDOD + \overset{\substack{O\\ \|}}{C}-\ CHDOH,\ \ \underset{\substack{\|\\ O}}{C}-CHDOD\ \ (4)$$

MATHEMATICS

Section – 1

1.(AC) Area of $\Delta ABD = \frac{1}{2} AB.AD.\sin\frac{A}{2}$

Area of $\Delta ACD = \frac{1}{2} AC.AD.\sin\frac{A}{2}$



$\therefore \quad \Delta = \text{Area of } \Delta ABC = \text{Area of } \Delta ABD + \text{Area of } \Delta ACD$

$$= \frac{1}{2} AD(AB+AC)\sin\frac{A}{2}$$

$$= \frac{1}{2} AD(c+b)\sin\frac{A}{2}$$

This gives

$$AD = \frac{2\Delta}{(b+c)\sin(A/2)}$$

$$= \frac{2\left[(1/2)bc\sin A\right]}{(b+c)\sin(A/2)}$$

$$= \frac{bc\left[2\sin(A/2)\cos(A/2)\right]}{(b+c)\sin(A/2)}$$

$$= \left(\frac{2bc}{b+c}\right)\cos\frac{A}{2}$$

Again, from ΔACD

$$\frac{AD}{\sin C} = \frac{AC}{\sin(180° - \theta)}$$

Where $\theta = \underline{|ADB}$. Hence

$$AD = \frac{b\sin C}{\sin\left[B + (A/2)\right]} \quad \left(\because B + \frac{A}{2} + \theta = 180°\right)$$

2.(BC)

$$8\sum_{k=1}^{\alpha}\cos^4\frac{k\pi}{2\alpha+1} = \sum_{k=1}^{\alpha}\left(3 + 4\cos\frac{2k\pi}{2\alpha+1} + \cos\frac{4k\pi}{2\alpha+1}\right)$$

$$\Rightarrow \quad 8 \times \frac{55}{16} = 3\alpha + 4\left(\frac{-1}{2}\right) - \frac{1}{2}$$

$$\Rightarrow \quad \alpha = 10$$

3.(BC)

$\left|z + \frac{1}{2}\right| \geq \frac{1}{2}$ for all $z \in S$

$$\left|z^2 + z + 1\right| = 1 = \left|\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}\right|$$

$$1 \leq \left|z + \frac{1}{2}\right|^2 + \frac{3}{4}$$

$$\left|z + \frac{1}{2}\right|^2 \geq \frac{1}{4}$$

$\left|z + \frac{1}{2}\right| \leq -\frac{1}{2}$ or $\left|z + \frac{1}{2}\right| \geq \frac{1}{2}$

Now $\left|\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}\right| = \left|z^2 + z + 1\right| = 1$

$$\left|\left|z + \frac{1}{2}\right|^2 - \frac{3}{4}\right| \leq 1$$

$$-1 \le \left|z+\frac{1}{2}\right|^2 - \frac{3}{4} \le 1$$

$$\frac{-1}{4} \le \left|z+\frac{1}{2}\right|^2 \le \frac{7}{4}$$

$$\left|z+\frac{1}{2}\right| \le \frac{\sqrt{7}}{2}$$

$$|z| \le \frac{\sqrt{7}}{2} + \frac{1}{2}$$

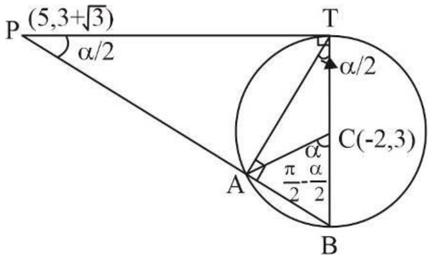
$$|z| \le \frac{\sqrt{7}+1}{2} \le \frac{4}{2}$$

$$|z| \le 2$$

4.(AC)

$$CT = CB = r = 4$$

$$PT = \sqrt{5^2 + \left(3+\sqrt{3}\right)^2 + 4\times 5 - 6\left(3+\sqrt{3}\right) - 3}$$



$$PT = 6$$

Let $\angle ACB = \alpha$

$$Ar(\Delta CAB) = \frac{1}{2} r.r\sin\alpha = 8\sin\alpha$$

$$Ar(\Delta CAT) = \frac{1}{2} r.r\sin(\pi-\alpha) = 8\sin\alpha$$

In ΔPBT $\tan\frac{\alpha}{2} = \frac{BT}{PT} = \frac{8}{6}$

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{2\times\frac{4}{3}}{1+\frac{16}{9}} = \frac{24}{25}$$

$$Ar(\Delta CAB) + Ar(\Delta CAT) = 16\sin\alpha = 16\times\frac{24}{25} = \frac{384}{25}$$

$$\lambda = 384$$

$$\text{Hence}\left[\sqrt{384}\right] - 15 = 19 - 15 = 4$$

5.(ABD)

Given, $\left|\hat{a}-\hat{b}\right|^2 + \left|\hat{b}-\hat{c}\right|^2 + \left|\hat{c}-\hat{a}\right|^2 = 9$

$$\Rightarrow \quad 2\left(a^2+b^2+c^2\right) - 2\left(\hat{a}.\hat{b} + \hat{b}.\hat{c} + \hat{c}.\hat{a}\right) = 9$$

$\Rightarrow \quad 6-2\left(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}\right)=9$

$\Rightarrow \quad 2\left(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}\right)=-3$

$\Rightarrow \quad \left(a^2+b^2+c^2\right)+2\left(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}\right)=3-3$

$\Rightarrow \quad |\hat{a}+\hat{b}+\hat{c}|^2=0$

$\Rightarrow \quad \left(\hat{a}+\hat{b}+\hat{c}\right)=0$

6.(AD)

$$\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$$

IF $=e^{\int \frac{2x}{1+x^2}dx}$

$=e^{\ell n\left(1+x^2\right)}=\left(1+x^2\right)$

$\therefore \quad y\left(1+x^2\right)=\int 4x^2dx=\frac{4x^3}{3}+C$

Passing through (0, 0) $\Rightarrow C=0$

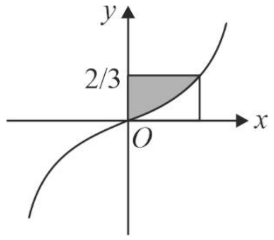
$\therefore \quad y=\frac{4x^3}{3\left(1+x^2\right)}$

$$\frac{dy}{dx}=\frac{4}{3}\left[\frac{\left(1+x^2\right)3x^2-x^3.2x}{\left(1+x^2\right)^2}\right]=\frac{4}{3}\left[\frac{3x^2+x^4}{\left(1+x^2\right)^2}\right]=\frac{4x^2\left(3+x^2\right)}{3\left(1+x^2\right)^2}$$

Hence, $\frac{dy}{dx}>0, \forall x \neq 0; \frac{dy}{dx}=0$ at $x=0$

And it does not change sign $\Rightarrow$ $x=0$ is the point of inflection $y=f(x)$ is increasing for all $x \in R$.

$x \to \infty, y \to \infty, x \to -\infty, y \to -\infty$



Area enclosed by $y=f^{-1}(x)$, x-axis and ordinate at $x=\frac{2}{3}$

$$A=\frac{2}{3}-\frac{4}{3}\int_0^1\frac{x^3}{1+x^2}dx$$

Put $1+x^2=t \Rightarrow 2xdx=dt$

$$A=\frac{2}{3}-\frac{2}{3}\int_1^2\frac{(t-1)}{t}dt$$

$$A=\frac{2}{3}-\frac{2}{3}\int_1^2\left(1-\frac{1}{t}\right)dt=\frac{2}{3}-\frac{2}{3}[t-\ell nt]_1^2=\frac{2}{3}-\frac{2}{3}\left[(2-\ell n2)-1\right]=\frac{2}{3}\ell n2$$

Section – 2

7.(B) 1 ball for box 2 and 1 for box 4 in ${}^7C_1\times{}^6C_1=42$ ways

Remaining 5 balls are to be put in 3 boxes = $3\times\left(2^5-2\right)+3=93$

Total = $42\times 93=3906$

8.(D) $YX=\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}6 & -5\\2 & 1\end{bmatrix}$ $\qquad XY^T=\begin{bmatrix}6 & -5\\2 & 1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$

$=\begin{bmatrix}10 & -3\end{bmatrix}$ $\qquad =\begin{bmatrix}-4\\4\end{bmatrix}$

$$Z=\begin{bmatrix}10 & -3\end{bmatrix}\begin{bmatrix}-4\\4\end{bmatrix}=\begin{bmatrix}-52\end{bmatrix}$$

$$P=\begin{bmatrix}-4\\4\end{bmatrix}_{2\times1}\begin{bmatrix}10 & -3\end{bmatrix}_{1\times2}=\begin{bmatrix}-40 & 12\\40 & -12\end{bmatrix}$$

$n(P)=4,\qquad |P|=480-480=0$

$tr(z)=-52$

$$\frac{n(P)\left(|p|^2+tr(z)\right)}{|z|}=\frac{4\times-52}{-52}=+4$$

9.(A) $A_1\rightarrow$ missing card is honour card

$A_2\rightarrow$ missing card is not honour card

$B\rightarrow$ 1st 4 card drawn consist only one honour card

$$P\left(\frac{A_1}{B}\right)=\frac{P(A_1)P\left(\frac{B}{A_1}\right)}{P(A_1)P\left(\frac{B}{A_1}\right)+P(A_2)P\left(\frac{B}{A_2}\right)}$$

$$=\frac{\left(\frac{4}{13}\right)\frac{{}^{15}C_1\times{}^{36}C_3}{{}^{51}C_4}}{\frac{4}{13}\frac{{}^{15}C_1\times{}^{36}C_3}{{}^{51}C_4}+\frac{9}{13}\times\frac{{}^{16}C_1\times{}^{35}C_3}{{}^{51}C_4}}=\frac{5}{16}$$

10.(B) $\ell na=\lim_{r\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}\ell n\left(2-\frac{r}{n}\right)=\int_0^1\ell n(2-x)\,dx=\ell n4-1\Rightarrow a=\frac{4}{e}$

$$\ell nb=\lim_{r\to\infty}\sum_{r=1}^{n}\frac{2r}{n^2}\ell n\left(1+\left(\frac{r}{n}\right)^2\right)=\int_0^1 2x\ell n\left(1+x^2\right)dx=\ell n\frac{4}{e}\Rightarrow b=\frac{4}{e}$$

Section – 3

1.(0) By the sum-to-product formulas, we have

$$\sin(x-y)+\sin(y-z)=2\sin\frac{x-z}{2}\cos\frac{x+z-2y}{2}$$

By the double angle formulas, we have $\sin(z-x)=2\sin\frac{z-x}{2}\cos\frac{z-x}{2}$.

Thus, $\sin(x-y)+\sin(y-z)+\sin(z-x)$

$$=2\sin\frac{x-z}{2}\left[\cos\frac{x+z-2y}{2}-\cos\frac{z-x}{2}\right]$$

$$=-4\sin\frac{x-z}{2}\sin\frac{z-y}{2}\sin\frac{x-y}{2}$$

$$=-4\sin\frac{x-y}{2}\sin\frac{y-z}{2}\sin\frac{z-x}{2}$$

By the sum-to-product formulas.

$$\left|\sin\left(\frac{x-y}{2}\right)\sin\left(\frac{y-z}{2}\right)\sin\left(\frac{z-x}{2}\right)\right|=\frac{1}{3}$$

2.(2) $\frac{dy}{dx}=\frac{y-2x^3}{x+2y\,x^2}$

$$x\,dy+2y\,x^2dy=y\,dx-2x^3dx$$

$$\frac{xdy-ydx}{x^2}=-\left(2ydy+2xdx\right)$$

$$d\left(\frac{y}{x}\right)=-d\left(y^2+x^2\right)$$

$$\frac{y}{x}=-y^2-x^2+k$$

$$P\left(x,y\right)=x^2+y^2+\frac{y}{x} \qquad \ldots\text{(i)}$$

As $P\left(1,1\right)=3$

$$\frac{x-1}{2}=\frac{y+1}{3}=1$$

$$x=3, y=2$$

$$\frac{c}{3}=9+4+\frac{2}{3}$$

$$\frac{c}{3}=\frac{41}{3}\Rightarrow c=41$$

3.(4) Let $I=\int_0^{\pi/2} e^{\sin x}dx+\int_1^e \sin^{-1}\left(\ell nx\right)dx$

Let $f\left(x\right)=e^{\sin x}; x\in\left[0,\frac{\pi}{2}\right]$

$\Rightarrow \quad f'(x)=\cos x.e^{\sin x}; x\in\left[0,\frac{\pi}{2}\right] \Rightarrow \quad f'(x)\geq 0 \forall x\in\left[0,\frac{\pi}{2}\right]$

Thus, $f\left(x\right)$ is injective function on $\left[0,\frac{\pi}{2}\right]$

Now $f\left(x\right)=e^{\sin x}\Rightarrow \ell nf\left(x\right)=\sin x$

$\Rightarrow \quad x=\sin^{-1}\left(\ell n\, f\left(x\right)\right)$

$\therefore \quad f^{-1}\left(x\right)=\sin^{-1}\left(\ell nx\right)$ when $x=0,\ f\left(x\right)=e^0=1$

And when $x=\frac{\pi}{2};\ f\left(x\right)=e^1=e$

$\therefore \quad$ (1) can be written as $I=\int_0^{\pi/2} f\left(x\right)dx+\int_1^e f^{-1}\left(x\right)dx=\frac{\pi}{2}\left(e\right)-\left(0\right)\left(1\right)=\frac{\pi}{2}e$

4.(1) $\log_{x^2+6x+8}\left(\log_{2x^2+2x+3}\left(x^2-2x\right)\right)=0$

$\therefore \quad \log_{2x^2+2x+3}\left(x^2-2x\right)=1$

$\therefore \quad x^2-2x=2x^2+2x+3$

$\Rightarrow \quad x^2+4x+3=0$

$\Rightarrow \quad (x+1)(x+3)=0$

$\therefore \quad x=-1,-3$

But for $x=-3, x^2+6x+8<0$

$\therefore \quad x=-1$

5.(6) $\alpha=\lim\limits_{x\to 0^+}\dfrac{e^{x^2}-\left(1-x^2\right)^{1/2}}{\sin^2 x}$

$$\alpha=\lim_{x\to 0^+}\frac{\left(1+x^2+\frac{x^4}{2}+.....\right)-\left(1-\frac{1}{2}x^2-\frac{1}{8}x^4....\right)}{\sin^2 x}$$

$\alpha=\dfrac{3}{2}$

6.(0) The characteristic equation of *A* is $|A-\lambda I|=0$

$\lambda^3-S_1\lambda^2+S_2\lambda-S_3=0$ where

$S_1=$ sum of the main diagonal elements $=2+1+2=5$

$S_2=$ Sum of the minors of main diagonal elements

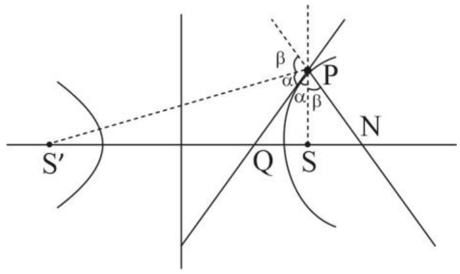
$$=\begin{vmatrix}1 & 0\\ 1 & 2\end{vmatrix}+\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}+\begin{vmatrix}2 & 1\\ 0 & 1\end{vmatrix}=(2-0)+(4-1)+(2-0)=2+3+2=7$$

$$S_3=|A|=\begin{vmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{vmatrix}=2(2-0)-1(0-0)+1(0-1)=4-1=3$$

$\therefore \quad$ The characteristic equation is $\lambda^3-5\lambda^2+7\lambda-3=0$

By C – H theorem, we get $A^3-5A^2+7A-3I=0$

7.(0)



$$\frac{SQ}{S'Q}=\frac{2}{3}$$

$$\frac{SN}{S'N}\times\frac{SP}{S'P}=\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$$

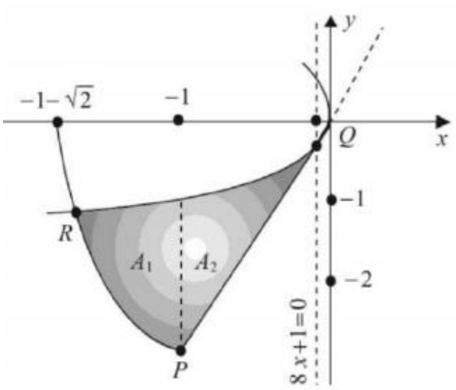
8.(1) Enforcing continuity at $x=1$ and $x=-1$ respectively gives

$$2=1+a+b,\ 1-a+b=-2$$

$$\Rightarrow \quad a=2, b=-1 \quad \Rightarrow \quad f(x)=\begin{cases}2x, & |x|\le 1\\ x^2+2x-1, & |x|>1\end{cases}$$

We now draw the graph of $f(x)$

R is given by $y^2=-\dfrac{x}{2}=\left(x^2+2x-1\right)^2 \Rightarrow \quad x=-2, y=-1 \quad \Rightarrow \quad R\equiv(-2,-1)$



Third quadrant is highlighted $Q\equiv\left(-\dfrac{1}{8},-\dfrac{1}{4}\right)$

The required area A is given by

$$A=A_1+A_2=\int_{-2}^{-1}\left(-\sqrt{\frac{-x}{2}}-\left(x^2+2x-1\right)\right)dx+\int_{-1}^{-1/8}\left(-\sqrt{\frac{-x}{2}}-2x\right)dx$$

$$=\int_{-2}^{-1/8}-\sqrt{\frac{-x}{2}}dx-\left\{\int_{-2}^{-1}\left(x^2+2x-1\right)dx+\int_{-1}^{-1/8}2x\,dx\right\}=-\frac{21}{16}-\left\{-\frac{5}{3}-\frac{63}{64}\right\}=\frac{257}{192}$$